

CHARACTERIZING SPATIAL STRUCTURE OF TREE CANOPY USING COLOUR PHOTOGRAPHS AND MATHEMATICAL MORPHOLOGY

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RÉSUMÉ

Dans cet article, les auteurs passent en revue un certain nombre de principes fondamentaux de transformation morphologique. Ils ont mis au point des algorithmes d'analyse granulométrique qu'ils ont ensuite utilisés pour caractériser trois espèces de jeunes arbres : le sapin, l'épinette et le pin à l'aide de clichés couleurs classiques. Les auteurs ont recouru à divers prétraitements pour séparer la cime des arbres de l'arrière-plan sur chaque cliché, ce qui a donné trois images binaires arbres-cimes qui ont fait l'objet, par la suite, d'un traitement morphologique. Deux types d'éléments structurants, soit des éléments linéaires et des éléments en forme de disque, ont été utilisés en vue de produire des signatures sur les structures, comme le spectre de patrons, à l'aide d'analyses granulométriques des images binaires des cimes d'arbre. Les résultats de l'étude montrent que les spectres de patrons de paramètres de distribution granulométrique extraits de chaque image binaire varient considérablement d'une espèce d'arbre à l'autre. Ainsi, les spectres des patrons structurels des cimes peuvent se révéler utiles pour la discrimination des espèces d'arbre.

SUMMARY

In this paper, some basic principles of morphological transformation are reviewed. Algorithms of granulometric analysis have been developed and applied to characterize three kinds of young trees — fir, spruce, and pine. Conventional colour photographs have been used. A number of preprocessing techniques have been used to separate the tree crown from the background in each photograph, resulting in three binary tree-crown images for subsequent morphological processing. Two types of structuring elements, linear and disk-shaped, have been used to generate structure signatures such as pattern spectra through granulometric analyses of the binary images of tree crowns. Experiment results indicate that pattern spectra of size distribution parameters extracted from each binary image are considerably different among the three tree species. Crown structural pattern spectra may therefore be useful alternatives for the recognition of tree species.

INTRODUCTION

Computer-based recognition of individual tree species from remotely sensed imagery requires a great deal of intelligence (Gougeon, 1993). Spectral properties alone are not sufficient for recognizing individual trees because a large number of tree species exhibit very similar spectral patterns. Features containing information on the spatial aspect of tree species provide further evidence for recognizing tree species.

Features of various targets recorded in remotely sensed images can be broadly divided into three groups: spectral signature, texture, and geometrical shape features. Spectral signature features are commonly used in optical remote sensing for the purpose of information extraction (see, for example, Asrar, 1989). Because spectral signature is best suited for per-pixel based operation in the detection, identification, and classification of targets and because a target

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very often occupies more than one pixel in an image, spectral signature features are usually used during the preliminary stages in object recognition tasks. Texture features referring to the spatial variation of grey levels within a predefined local neighbourhood or kernel of an image have been used in remote sensing classification (see, for example, Haralick *et al.*, 1973). Texture features are sometimes used as ancillary data in combination with spectral data in image classification (Marceau *et al.*, 1990; Gong *et al.*, 1992).

Geometrical shape features refer to parameters describing the spatial structure and shape of individual objects, each consisting of hundreds of pixels in an image. Thus, geometrical shape features are often used during the advanced stages of object recognition (see, for example, Gong and Howarth, 1990; Nagao and Matsuyama, 1980). They can be further divided into two subgroups: external and internal (Pitas and Venetsanopoulos, 1992). External representations are based on the description of the geometrical boundary. Fourier descriptors and B-spline representation are two commonly used methods. The feature parameters for internal representations are based mainly on octrees, quadtrees, skeletons, and shape decomposition. Primarily because of the irregular shapes, which require complicated computation, external features are rarely used in remote sensing application.

Many algorithms have been proposed to represent the internal features (Matheron, 1975; Serra, 1982; 1986; Lee *et al.*, 1986; Sternberg, 1986; Destival, 1986; O'Brien, 1989; Vogt, 1989; Zheng and Zhou, 1992). The morphological skeleton and shape decomposition are two of the most commonly used. Because the skeleton operation is highly sensitive to noise, it is most suitable for describing simple and regular objects in a filtered image. Shape decomposition based on mathematical morphology is a general and effective approach for describing the geometric shape of various targets. The concept of measuring the successive results of the morphological openings on an image by different sized structuring elements was initially suggested by Matheron (1975) and Serra (1982, 1986). Matheron provided a basic representation theorem about granulometric size distributions. Intuitively, a binary image is referred to as a series of grains, and the grains are sieved through morphological opening operations with increasing size of structuring elements. These residual areas of the filtered image form a size distribution that is indicative of image structure. A full discussion of granulometric analysis and its relation to sieving can be found in other studies (see, for example, Giardina and Dougherty, 1989; Sand and Dougherty, 1992; Dougherty *et al.*, 1992; Shih and Pu, 1992).

The objective of this paper is to present a new application of shape decomposition through morphological operations in extracting geometric features in tree canopy images for identifying three different types of young conifer trees. Based on the idea of granulometric analysis, we developed a method using linear structure elements of different directions to probe the structural size distribution of a canopy. Experimental results showing the effectiveness of the approach for extracting shape signatures of tree canopies are presented.

MATHEMATICAL MORPHOLOGY

Mathematical morphology provides an approach to the processing of digital images that is based on geometric shape. It uses set operators such as union, intersection, and complementation (Flouzat, 1989). Properly used, mathematical morphological operations tend to simplify image data, preserving their essential shape characteristics and eliminating irrelevancies (Haralick *et al.*, 1987). Different operations in mathematical morphology are generated by a structuring element — a particular shape of pixels — acting on the objects independent of the size, shape, and grey level. When acting on complex shapes, the compositions of the operators are such that they are able to decompose the shapes into their meaningful parts and to separate the meaningful parts from their extraneous parts. Such a system of operators and their compositions permit the underlying shapes to be identified and reconstructed as best as possible from their distorted noisy forms. In addition, they permit each shape to be understood in terms of its decomposition, since each part of the decomposition is relatively simple.

Structuring Elements

In mathematical morphology, structuring elements are considered to be probes or special tools for looking at images (Pratt, 1991). The simplest structuring element is a single point. If the point is on the origin, we have an identity transformation, or exact copy of the original image. A line structuring element consists of a contiguous linear sequence of points, with the origin at the middle. Since translation commutes with Minkowski Add (Minkadd) and Minkowski Subtraction (Minksub), placing the origin somewhere else would only modify the result by a translation. The line structuring element has a length and an orientation, and is clearly used to locate linear structures, particularly those with fixed orientations, less than a certain length. If linear structures of unknown orientation are to be found, a union of lines at different orientations can be used. Directional filtering belongs to this processing family.

Square, hexagon, and octagon are some homothetic structuring elements. The hexagonal grid has an advantage over the square one, since a hexagon is much closer in shape to a disk than a square; hence, it is less sensitive to direction. Larger structuring elements can be decomposed into smaller ones. Thus, the equivalent of a complex structuring element can be achieved by applying a sequence of simple ones with a reduced computation requirement (Haralick *et al.*, 1987). Decomposing complex structuring elements is based on the following four important identities, two for dilation/Minkowski addition, and two for erosion/Minkowski subtraction:

- Union Decomposition:

$$(X \oplus (S_1 \cup S_2)) = ((X \oplus S_1) \cup (X \oplus S_2)) \quad (1)$$

$$(X \ominus (S_1 \cup S_2)) = ((X \ominus S_1) \cap (X \ominus S_2)); \quad (2)$$

- Dilation Decomposition:

$$(X \oplus (S_1 \oplus S_2)) = ((X \oplus S_1) \oplus S_2) \quad (3)$$

$$(X \ominus (S_1 \oplus S_2)) = ((X \ominus S_1) \ominus S_2). \quad (4)$$

Equations 1 and 2 allow one to decompose a Minkadd or Minksub operation by any large structuring element into a union of Minkadds or intersection of Minksubs by smaller structuring elements whose union makes up the larger one. This 'union decomposition' is akin to breaking a large number into a 'sum' of smaller components. Equations 3 and 4 allow one to decompose the dilation or erosion by certain large structuring elements (those that can be constructed by the Minkowski addition of smaller ones) into a sequence of successive dilation's or erosions, each step operating on the results of the last one, simplifying the computation complexity (Zhuang and Haralick, 1986).

Morphological Transformation and Operators

All morphological transformations are non-reversible. For recognition and classification, only the signatures that can represent the essential shape and structure are extracted through morphological transformations. Two basic operations are dilation and erosion, defined as Minkadd and Minksub, respectively.

Dilation, $D(A, S)$ is defined as:

$$D(A, S) = \bigcup_{s \in S} A + s \quad (5)$$

where A is the image and S is the structuring element; s is an element in S . Dilation has the effect of "expanding" an image object.

Minkowski difference is defined as:

$$A \ominus S = \bigcap_{s \in S} A + s. \quad (6)$$

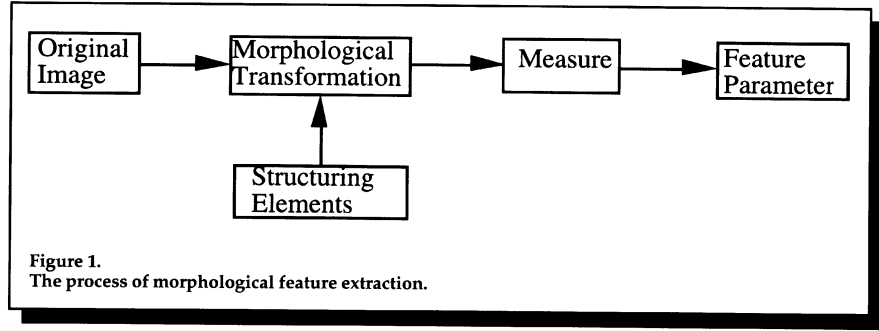
In Equations 5 and 6, A is translated by every element of S and then the union and intersection are taken, respectively. Erosion of A by S is defined as $E(A, S) = A \ominus (-S)$. Eroding an image by a structuring element S has the effect of "shrinking" the image in a manner determined by S . If $-S = S$, that is, if S is symmetric, we have $E(A, S) = A \ominus (S)$. Based on these two operations, opening and closing are defined. They play a central role in image morphological analysis.

Opening is defined as:

$$O(A, S) = [A \ominus (-S)] \oplus S = D[E(A, S), S]. \quad (7)$$

Closing is defined as:

$$C(A, S) = [A \oplus (-S)] \ominus S = E[D(A, -S), -S]. \quad (8)$$



Morphological Feature Extraction

Spectral signatures and textual features are two fundamental types of information for classifying image pixels into different classes. In many applications of computer vision, we need to identify an object, which may consist of many pixels, as a whole. Characteristic parameters such as the size and shape of an object are necessary for representation and recognition of the object. Shape representation and recognition of objects require the transformation or preprocessing of an original image into a series of points or curves that can reflect the geometric properties of the object.

For object identification, we wish to extract new parameters from the preprocessed image that can effectively represent different objects. The information revealed by the new parameters is completely determined by the feature extraction procedure. In this study, the feature extraction is realized by applying morphological operations to the preprocessed image with different structuring elements. The procedures of morphological feature extraction used in this study are shown in Figure 1.

GRANULOMETRIES AND PATTERN SPECTRA

A pattern spectrum generated from morphological transformation with a family of structure elements reflects the shape and size characteristics of the object. The idea is to apply opening operation to decompose an image through a series of structuring elements with a specific shape. The opened images are compared with the original preprocessed image to generate measures with respect to different sizes of structuring elements with the same shape. These measures can be used as shape signatures of the preprocessed image and can be plotted as a pattern spectrum. Different shapes of structure elements can generate different pattern spectra.

Let $\{S(n) \mid n = 1, 2, \dots, N\}$ be a series of structuring elements, and $S(n) = n S_0$, S_0 is a structuring element that includes the origin. A is the preprocessed image. $\{A(n) \mid n = 1, 2, \dots, N\}$ is the sequence of images being opened, where $A(0) = O(A, S_0)$. $A(n) = O(A, n S_0)$, $n = 1, 2, \dots, N$. According to the properties of opening operation, if $n \geq m$, then $A(n) \subset A(m)$. Therefore, the sequence $\{A(n) \mid n = 1, 2, \dots, N\}$, a set of images with object sizes decreasing as the structuring

element gets larger, provides shape and size information out the image A .

Let $CARD[A(n)]$ be the measure of the cardinality of $A(n)$. Because $A \supset A(0) \supset A(1) \supset A(2) \supset \dots \supset A(N)$, the following relations hold:

$$CARD[A] \geq CARD[A(0)] \geq CARD[A(1)] \geq \dots \geq CARD[A(N)]. \quad (9)$$

As long as S_0 consists of more than a single point and K is large enough, ($1 < K < N$), then

$$CARD[A] \geq CARD[A(0)] \geq CARD[A(1)] \geq \dots \geq CARD[A(K-1)] \geq CARD[K] = CARD[A(K+1)] = \dots = CARD[A(N)] = 0. \quad (10)$$

The mapping $n \rightarrow O(A, nS_0)$ is called a granulometry, and the function $CARD[A(n)]$ is known as the size distribution generated by the granulometry (Dougherty *et al.*, 1992).

Although $CARD[A(n)]$ is increasing and translationally invariant, it is not rotationally invariant unless the structure element S_0 is a closed unit disk. Therefore, granulometric size distributions are sensitive to orientation. This property may be used to extract the directional texture structure in an image with linear structuring elements.

Different shapes of image objects can lead to different size distributions when opened by the same structuring element. Different size distributions may be obtained from opening with different directional structuring elements to the same image. As long as the objects are not absolutely symmetric, when a family of structuring elements with the same direction but different sizes are used, different geometric properties of the image can be revealed. Therefore, should an image contain intricate texture and shape properties, they would be revealed by granulometric analysis and represented by pattern spectra.

To eliminate the effect of different unit to size distribution, it is necessary to normalize the size distribution $CARD[A(n)]$ by introducing the function

$$F(n) = 1 - \frac{CARD[A(n)]}{CARD[A]}. \quad (11)$$

It is clear that $0 \leq F(n) \leq 1$ and $F(n)$ can be regarded as a probability distribution function. Its derivative $F'(n)$ is a probability density function. It has become popular to refer to the normalized granulometric size distribution density as the pattern spectrum of the image (Sand and Dougherty, 1992). Using $F'(n)$, we can compute the moments:

$$m^k = \int_0^K n^k F'(n) dn \quad (12)$$

where we assume that $n = K$ is the point where $F'(n)$ vanishes. The moments m^k can be employed as feature parameters.

Granulometric size distributions can be employed to study shape-size complexity, multiscale shape representation, and symbolic image modelling, which can be used as signature parameters for pattern recognition and classification.

EXPERIMENT ON CHARACTERIZING YOUNG CONIFER TREES

An experiment was designed and conducted to extract the crown shape parameters from photographs of three young trees, fir (Plate 1a), spruce, and pine (Plates 2a and 2b, respectively). The purpose was to test whether the size distribution parameters obtained from these three young tree species can properly distinguish these species. The photographs were taken at Petawawa National Forest Institute during a clear day in early June 1991. To evaluate the effectiveness of pattern spectrum for representing crown shape, some preprocessing is needed to separate the tree canopy from the background. This requires image

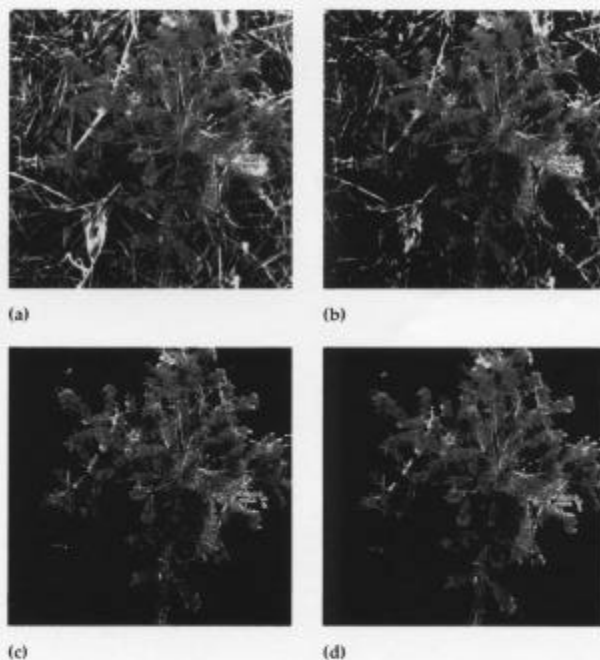


Plate 1. The original colour photograph of a young fir tree (a); clustering results (b); contextual adaptive filtering results (c); and morphologically filtered results (d). The background noise has been gradually suppressed. (d) was used as an binary image for the granulometric analysis.

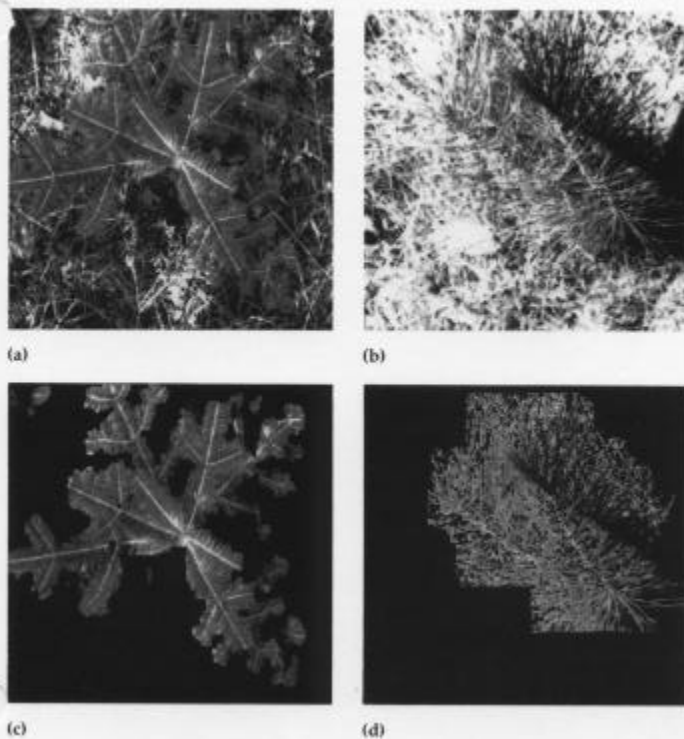


Plate 2.
The original colour photographs of a young spruce tree (a) and a young pine tree (b). The post-processed images of the spruce (c) and the pine (d) were used in the granulometric analysis.

segmentation with some noise reduction, such as contextual and morphological filtering. Shape parameters can then be extracted through granulometric analysis. The processing procedures are shown in Figure 2.

Image Segmentation

A K-means clustering technique was applied to segment the original image into two classes, tree and background, based only on the spectral information of the image. Because of the spectral signature confusion between the tree and the background, the "tree crown" cannot be completely separated from the background. Too much noise appears in the binary image of the tree crown (see, for example, Plate 1b).

Contextual Filtering

To eliminate the background noise, contextual information was considered. A window was used to find the noise points to be removed based on the following adaptive filtering technique.

For a binary image $f(x, y)$, construct two parameter matrices:

$$t_b(x, y) = \begin{cases} 1 & \text{if the class of } f(x, y) \text{ is branch} \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

$$t_l(x, y) = \begin{cases} 1 & \text{if the class of } f(x, y) \text{ is leaf} \\ 0 & \text{otherwise} \end{cases} \quad (14)$$

$$f_b(x, y) = \sum_{t_b(m, n) \in W} t_b(m, n) \quad (15)$$

$$f_l(x, y) = \sum_{t_l(m, n) \in W} t_l(m, n) \quad (16)$$

$$g(x, y) = \begin{cases} 1 & \text{if } f(x, y) = 1 \text{ and } (f_b(x, y) > T_b \text{ and } f_l(x, y) > T_l) \\ 0 & \text{otherwise} \end{cases} \quad (17)$$

where w is a kernel centred on (x, y) . After some experiments, an 11 by 11 neighbourhood was selected for this study. $0 < T_b$ and $T_l < 11 \times 11$ are thresholds empirically determined; $g(x, y)$ is the enhanced image (see, for example, Plate 1c).

Morphological Filtering

Morphological filters were used to further reduce noise in the images. Structuring elements were carefully selected. Opening operations helped to reduce external noise, while closing operations helped to reduce internal noise. The improvement in noise reduction can be observed by comparing Plate 1c with Plate 1d. In Plate 1d, much background noise has been removed from Plate 1c.

Shape Signature Extraction

With a family of structuring elements of the same shape, we can sieve the image through a series of opening operations to generate a pattern spectrum for each tree canopy. In general, to probe the pattern spectrum for a specific shape, such as circles with different sizes, the family of structuring elements with similar shapes (not exactly the same because they are rasterized versions) should be used to sieve the image. Because there is no regular grain for a young tree canopy, we designed two types of structuring elements, lines and disks. A series of linear structuring elements with eight directions ($0^\circ, 22.5^\circ, 45^\circ, 67.5^\circ, 90^\circ, 112.5^\circ, 135^\circ, 157.5^\circ$) were applied to calculate the size distributions. For every direction, the length of the linear structuring element was gradually increased during the morphological opening process until each tree crown was completely sieved through with no crown residuals on the resultant image. For a structuring element of a particular size and direction, a parameter was obtained through dividing the tree crown area in the filtered image by that in the original binary image (the division part in Equation 11). For each direction, a parameter vector was thus obtained, with each vector element corresponding to a specific size of a linear structuring element. Such a vector is the size distribution, as mentioned in the "Introduction." To compare the effects of linear structuring elements of different directions,

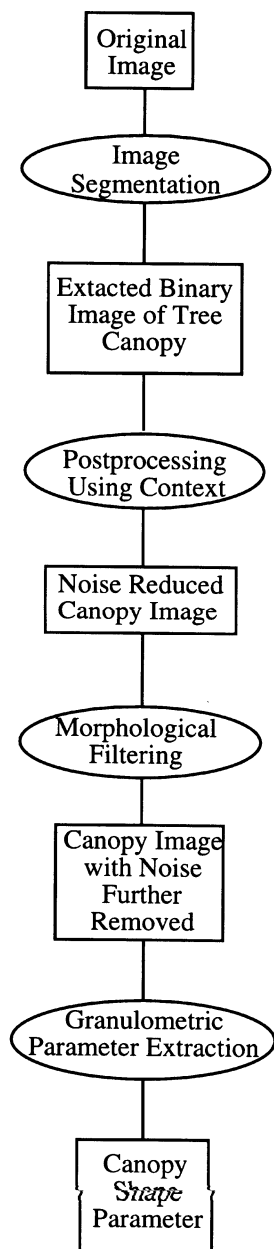


Figure 2.
Procedures for characterizing young conifer trees.

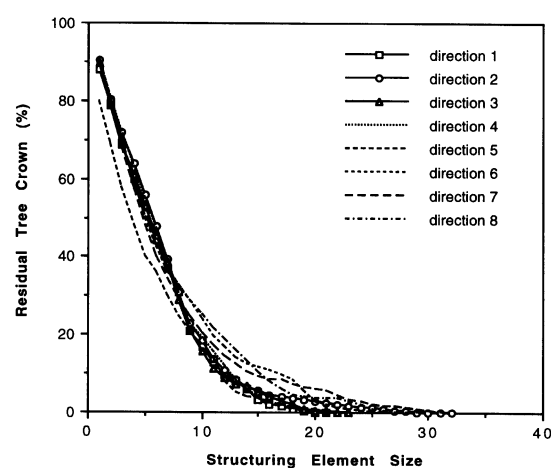


Figure 3.
Size distribution of young fir tree crown obtained using linear structuring elements.

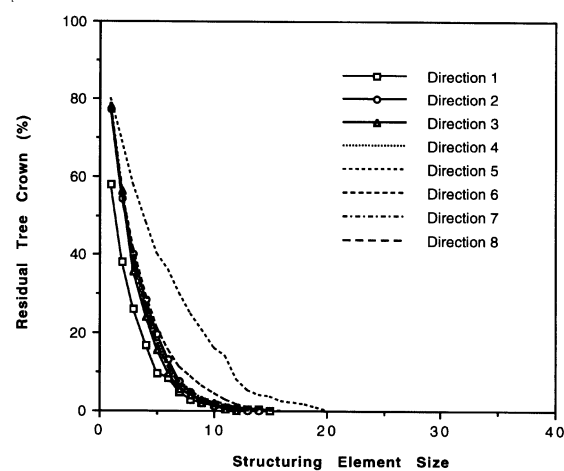


Figure 4.
Size distribution of young spruce tree crown obtained using linear structuring elements.

interpolation was applied to ensure that the length units among different directions were the same. Size distribution parameters can be tabulated or graphically plotted. Figures 3, 4, and 5 were obtained from the processing of the fir, spruce, and pine image, respectively. It can be seen from those figures that as the direction of linear structuring elements changes, the pattern spectra are different. This indicates that the distribution of tree branches and leaves is directional.

A series of different sized disk-shaped structuring elements, which were translation and rotation invariant, were simulated. They were then applied to the three binary images. The morphological filtering results are presented in Figure 6 using the size distribution parameters. It is clear from Figure 6 that the size distribution parameters are different among the three tree species. This is also supported by

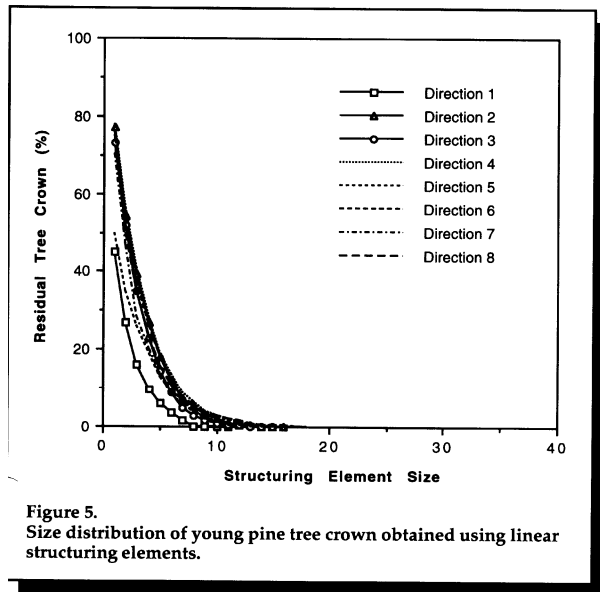


Figure 5.
Size distribution of young pine tree crown obtained using linear structuring elements.

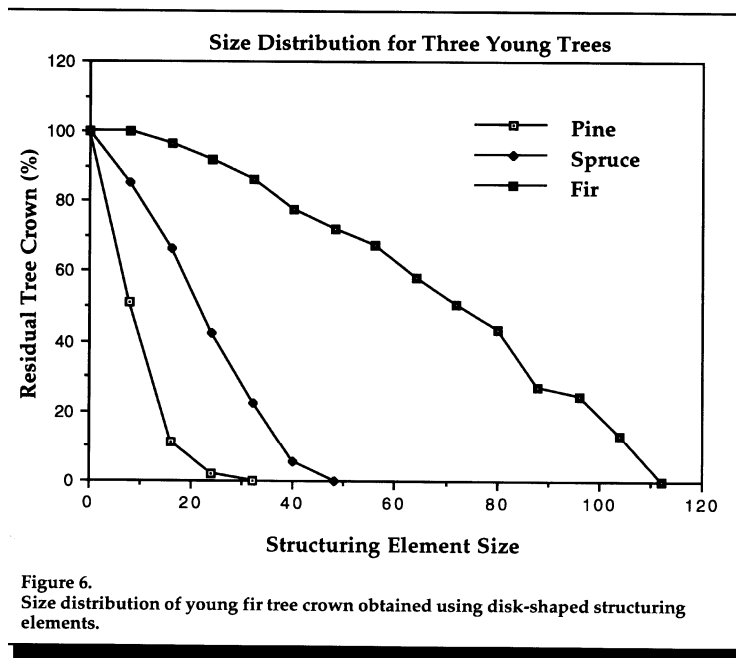


Figure 6.
Size distribution of young fir tree crown obtained using disk-shaped structuring elements.

Figures 3, 4, and 5. Therefore, size distribution parameters are effective in characterizing these three tree canopies.

SOME PROBLEMS AND DISCUSSION

Preprocessing

Our experiment is based on the assumption that the tree crown can be completely separated from the background based on the spectral and contextual information, which may not be possible using the simple clustering technique of K-means in practical application. Advanced algorithms may be needed for separating individual tree canopies from their background.

Shape of Structure Elements

If the scale of images is large enough to allow tree leaves to be observed, structuring elements with shapes similar to the various types of leaves may be useful in generating distinct shape signatures.

Orientation

The distribution of a tree canopy may have a specific direction in a specific ecological environment, which makes it possible to use directional structuring elements. We used eight directions to generate canopy pattern spectra. In some cases, linear structuring elements with 16 directions may provide more information about the shape of a crown and the density of the branches and leaves.

Morphological Texture

Texture information can be obtained from local size distribution, in which only pixel windows rather than the entire image are used to measure the size distribution. A local pattern spectrum can be produced for each pixel. In pixel classification, a homogeneous area having similar texture should have similar pattern spectra. Local pattern spectra may therefore be used to improve the classification accuracy.

Tree Recognition

We only took three images of young conifer trees in our experiments, and the differences of the geometric parameters and pattern spectra for the trees are considerably large. Further work is necessary to check if the same kind of trees produce similar geometric parameters.

CONCLUSIONS

Morphological transformation is an effective approach for revealing the structure and texture information of certain features in an image. Structuring elements play a key role in morphological transformation

and filtering. Pattern spectra based on size distribution parameters are different for the three different kinds of trees studied here, reflecting the difference in structure of these trees. Should an image contain intricate texture properties, they would be revealed by granulometric analysis. Linear structuring elements with different directions can probe the distribution and shape of a tree canopy. The differences between pattern spectra from size-distribution parameters with linear structuring elements are significant. Morphological operations based on disk-shaped structuring elements seem to be sufficient for differentiating the young fir, spruce, and pine trees.

To conduct granulometric analysis, the colour photographs must be preprocessed to convert the colour photographs into binary images. This was achieved in this study through clustering, contextual adaptive filtering, and morphological filtering. During the contextual filtering and the morphological filtering stages, a large amount of analyst interaction was required. To improve the processing efficiency, further research is required to evaluate alternative preprocessing strategies. Further study should also be made to reduce the amount of computation required in extracting structural features based on the morphological operations. Decomposing the structuring element during the morphological operation needs to be tested (Zhuang and Haralick, 1986; Boomgaard and Balen, 1992). Classification algorithms may be applied to classify tree species based on size distribution parameters of tree crowns. The role of different image resolutions should be investigated for close-range ground-based tree species recognition because in ground-based photographs the scale of trees at different distances from the camera changes to a significant degree.

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